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SIMULATION OF PARTICLE-TO-PARTICLE INTERACTIONS IN GAS–SOLID FLOWS

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Abstract—A Lagrangian simulation technique of a non-dilute gas–solid suspension flow is presented. The method is based on the introduction of simulated particle-to-particle collisions during the trajectory calculation of a particle, with a probability which is predicted through an iterative process, since it depends on the local concentration and velocities. This simulation makes it possible to predict dense gas–solid flows, without having to compute simultaneous trajectories. After a description of the simulation principle, we examine the dynamics of a binary collison between spherical particles, including rotation effects. As a test case, a horizontal gas–solid pipe flow was investigated at loading ratios up to 20. Results concerning velocity and concentration profiles, as well as pressure losses, are presented. Comparisons are made with the case of dilute flow simulation, revealing that particle-to-particle interactions play a non-negligible role as soon as the loading ratio exceeds unity.

Key Words: two-phase flow, gas–solid flow, Lagrangian simulation, particle collisions, pressure drop

1. INTRODUCTION

Many industrial applications of gas–solid flows, such as pneumatic conveying, involve high or moderately high concentrations of the particulate phase. For this kind of two-phase flow, approaches based on the dilute suspension hypothesis do not allow the influence of the collisions between particles to be predicted. As suggested by Pai (1974), the effect of particle-to-particle collisions can be taken into account by means of an analogy with the kinetic theory of gases and the derivation of the Boltzmann equation for the solid particles: such a statistical approach was described recently by Kitron *et al.* (1990), who used a simple model for the interparticle collision dynamics (Jenkins & Savage 1983). A more elaborate collision model, including the effect of particle rotation, was used by Lun & Savage (1987), who examined the case of a Couette flow by the kinetic theory.

The present paper describes a method of taking into account the particle-to-particle interactions in the Lagrangian approach of a gas–solid flow. Lagrangian simulations, which are widely used for the predictions of suspension flows, consist of the computation of a large number of individual particle trajectories. They are generally limited to the case of dilute suspensions, where the role of interparticle collisions is negligible. Classical Lagrangian simulation codes can be classified as follows:

- *Eulerian–Lagrangian formulation.* The influence of the particles on the fluid phase is taken into account by introducing source terms in the Eulerian equations governing the fluid flow [see, for instance, Tsuji *et al.* (1987)]. In such simulations, which require an iterative process for the mutual interphase coupling to be simulated, and are known as "two-way" simulations, special attention is often given to the dispersion of small particles in a turbulent fluid flow, which can be calculated using a modified $k-\epsilon$ or algebraic stress model (Durst *et al.* 1984; Weber *et al.* 1984; Berlemont & Desjonquères 1987; Milojevic 1987; Berlemont *et al.* 1990; Sommerfeld & Krebs 1990).

- *One-way coupling.* Particle trajectories are computed in a known fluid flow, i.e. the influence of the particles on the fluid is neglected. Two- and three-dimensional studies have been published (e.g. Matsumoto *et al.* 1976; Tsuji & Morikawa 1978;

Tsuji *et al.* 1985; Oesterlé 1987; Govan *et al.* 1989). Provided the loading is not too high, this kind of simulation is more suited for coarse particles, which have such high inertia that their motion is barely influenced by fluid turbulence.

The numerical prediction described in this paper, which concerns suspension flows of coarse particles, was obtained by one-way simulation. At moderate loading, the use of a one-way simulation is justified for such particles, insofar as slight modifications in the fluid velocity profile do not significantly alter the results if the particles are not too fine. Furthermore, the main objective of this work is the simulation of particle-to-particle collisions, which can be explained, without loss of generality, in the framework of a one-way Lagrangian simulation.

Although the response of coarse particles to gas turbulence is very weak, it was found that the role of the fluid fluctuations is not always negligible in the mechanism of suspension. Therefore, the influence of turbulence was simulated by a simple method which is valid in the case of small particle velocity fluctuations.

Existing simulations of dense flows are based on the simultaneous computation of several trajectories, in order to predict the flow of a particle cluster. Such a simulation was first proposed by Ottjes (1978). A two-dimensional granular shear flow with cylindrical particles was simulated by Campbell & Brennen (1985), and a recent example of application to plug flow in a pipe was given by Tsuji *et al.* (1990), who calculated the simultaneous displacements and mutual interactions of 1000 particles. Because the number of particles is limited by memory size, such methods cannot be used to predict the suspension flows of small particles over a long distance. It can be shown that the number of particle trajectories which have to be computed simultaneously in order to provide a realistic representation of a gas–solid pipe flow (for instance) is proportional to the loading ratio and to the pipe volume, and inversely proportional to the mass of a particle. As a matter of fact, the number of particles passing the inlet section of the pipe per unit time is

$$\dot{n} = \frac{\dot{m}_p}{m}, \qquad [1]$$

where $\dot{m}_p$ is the mass flow rate of the particulate phase and m is the mass of a particle. In order that the whole pipe, of length L, be occupied by the solid phase flowing at mean velocity V_p, such a simulation would require that trajectories be calculated during a (simulated) time T which can be estimated by L/V_p. Thus, the minimal number of simultaneously computed trajectories is

$$\dot{n}T = \frac{\dot{m}_p L}{m V_p} = \frac{\dot{m}_p}{\dot{m}_f}\frac{V_f}{V_p}\frac{\rho_f A L}{m}, \qquad [2]$$

with V_f, ρ_f and $\dot{m}_f$, denoting, respectively, the mean velocity, the density and the mass flow rate of the fluid, and A denoting the pipe section area.

Even at moderate loading ratios, this number is of the same order of magnitude as the ratio of the total fluid mass $\rho_f AL$ to the mass m of a particle. It can be concluded that in practical cases of pneumatic transport, this number is always much too large to permit the use of such a simultaneous Lagrangian simulation method.

In this work, we present a new technique which makes it possible to simulate moderately dense suspension flows by computing a reasonable number of successive individual particle trajectories. The principle consists of introducing artificial interparticle collisions during the trajectory calculation, the probability of these collisions depending on the local concentration and velocity of the solid phase. After a brief description, in section 2, of the computation of an individual trajectory between two collisions, which is somewhat classical, the process of collision simulation will be detailed in section 3, which includes the study of the dynamics of a binary collision between spherical translating and rotating particles. The main advantage of our method lies in the fact that an increase in the loading ratio does not require an increase in the number of trajectories. Note that two different loading ratios will not yield similar results (i.e. velocity and concentration profiles will not have the same shape), because the number of collisions is proportional to the square of the concentration.

A gas–solid flow in a 30 mm dia horizontal straight pipe, with a loading ratio upto 20, was numerically predicted using this technique. Particles were glass beads with dia 0.1 mm. Results

concerning velocity and concentration distributions over the height of the pipe are given herein, and are compared with the case of dilute flow simulation (i.e. without particle-to-particle interactions). The effect of particle-to-particle interactions on the concentration distributions is found to be particularly pronounced. The comparison between predicted pressure losses in both cases (with and without particle-to-particle interactions) and experimental results or existing correlations is satisfactory. Despite the lack of available experimental information in the range of high loading ratios, it could be concluded that particle-to-particle interactions must be taken into account in any gas–solid flow simulation with a loading ratio exceeding unity.

2. COMPUTATION OF PARTICLE TRAJECTORIES IN BOUNDED GAS–SOLID FLOWS

Between two particle-to-particle or particle-to-wall collisions, the three-dimensional motion of a spherical particle is described by a set of six scalar equations, expressing the linear and angular momentum conservation. The corresponding vectorial equations are:

$$m\frac{d\mathbf{V}_P}{dt} = m\mathbf{g} + \mathbf{D} + \mathbf{L}, \qquad [3]$$

with

$$\mathbf{D} = \tfrac{1}{2}\rho_f \mathbf{V}_r^2 \frac{\pi d^2}{4} C_D \frac{\mathbf{V}_r}{\|\mathbf{V}_r\|} \qquad [4]$$

and

$$\mathbf{L} = \tfrac{1}{2}\rho_f \mathbf{V}_r^2 \frac{\pi d^2}{4} C_L \frac{\boldsymbol{\Omega}_r \times \mathbf{V}_r}{\|\boldsymbol{\Omega}_r\| \cdot \|\mathbf{V}_r\|}, \qquad [5]$$

and

$$\frac{md^2}{10}\frac{d\boldsymbol{\Omega}_P}{dt} = -\pi\mu\, d^3 \boldsymbol{\Omega}_r, \qquad [6]$$

where $\mathbf{V}_P$ and $\boldsymbol{\Omega}_P$ are the linear and angular velocities of the particle, $\mathbf{V}_r$ is the translational relative velocity, $\boldsymbol{\Omega}_r$ is the angular relative velocity component in the plane orthogonal to $\mathbf{V}_r$, $\mathbf{g}$ is the gravitational acceleration. $\mathbf{D}$ is the drag force, $\mathbf{L}$ is the lift force, C_D and C_L are the drag and lift coefficients, d is the particle diameter and ρ_f is the fluid density.

The drag coefficient is expressed by the correlation of Morsi & Alexander (1972). The lift force acting on a spherical rotating particle was theoretically derived by Rubinow & Keller (1961), under the assumption of very small Reynolds numbers for translation and rotation. If the particle Reynolds number (Re) is of the order of 10 (as in our numerical tests), an empirically modified expression of the lift can be used, according to [5]: the direction of the lift force is the same as predicted by Rubinow & Keller (1961). The lift coefficient can be estimated by (Oesterlé *et al.* 1991):

$$C_L = (0.35 \pm 0.10)\frac{\|\boldsymbol{\Omega}_r\| d}{\|\mathbf{V}_r\|}. \qquad [7]$$

The hydrodynamic torque on the r.h.s. of [6] was first given by Stokes, and extended by Faxen in the case of a non-uniform fluid flow. Experimental results concerning the torque, obtained by Sawatzki (1970) in the case of non-translating spheres, reveal that this Faxen formula remains valid at moderate Re (based on rotational velocity). It is expected here that this torque is not affected by the translation when the particle Re is of the order of 10.

Note that the lift force and the hydrodynamic torque are taken into account because of the high rotational velocity induced by collisions.

The trajectory computation, which is numerically performed using a Runge–Kutta method, is reinitialized after each collision with the wall or with another particle. The latter case will be examined in the next section. When a particle-to-wall collision occurs, the new linear and angular velocity components are calculated in terms of the components just before impact through the

linear and angular momentum conservation, and by means of the classical coefficient of restitution. Friction and rotation are taken into account, leading to an asymmetric bounce, although particles are assumed spherical. The expressions for the linear and angular velocity components after impaction, which are different whether the particle slides on the wall or not, were given by Tsuji *et al.* (1987). The sliding condition is obtained by application of Coulomb's law, assuming given coefficients of static and kinetic friction. After each particle-to-wall collision, the axial momentum loss of the particle is stored with a view to the prediction of the additional pressure drop, as described in section 4.

This method of trajectory calculation has been proved efficient by yielding results which are in close agreement with experience in the case of dilute pipe flows (Oesterlé 1987), without introducing any other random process, like an irregular bouncing model, for particle-to-wall collisions (Matsumoto *et al.* 1976; Tsuji *et al.* 1985), providing that the turbulence effects are taken into account even if the particle are coarse.

During the motion of the particle, the influence of turbulence is simulated approximately in the following manner, based on an extension of the well-known Tchen theory (e.g. Hinze 1975), which leads, in the case of solid particles in a gas, to a simple estimation of the ratio between particle and fluid energy spectra:

$$\frac{E_p(\omega)}{E_f(\omega)} = \frac{1}{1+\omega^2 t^{*2}}, \qquad [8]$$

where ω denotes the angular frequency and t^* is the particle relaxation time, assumed constant in the derivation of Tchen. In our case, the relaxation time is not constant because particles are not supposed to obey Stokes' law. Nevertheless, this relaxation time can be derived at each trajectory location from the drag correlation of Morsi & Alexander (1972). According to Hinze (1975), an approximate expression for the normalized turbulence energy spectrum reads:

$$E_f(\omega) = \frac{4\mathfrak{I}}{1+\omega^2\mathfrak{I}^2}, \qquad [9]$$

where $\mathfrak{I}$ is an integral time scale which depends on the reference frame. In order to take crossing trajectory effects into account, the fluid energy spectrum must correspond to the fluctuations experienced by a particle along the particle path. If the Eulerian integral time scale is known (for transverse fluctuations in a pipe flow, one can assume $\mathfrak{I} \cong 0.1\, R/V_f$), it is possible to deduce an approximate expression for the fluid energy spectrum in the particle Lagrangian reference frame, according to the frequency translation proposed by Rogers & Eaton (1990). The ratio of particle to fluid mean square velocity fluctuations, which is given by

$$\frac{\overline{v_p'^2}}{\overline{v_f'^2}} = \frac{\int_0^\infty E_p(\omega)\,d\omega}{\int_0^\infty E_f(\omega)\,d\omega}, \qquad [10]$$

can thus be estimated, and the corresponding particle velocity fluctuations can be generated numerically during the calculation of an individual particle trajectory.

At each time step in the trajectory computation, the velocity components of the particle are modified by adding such a numerically generated random fluctuation, according to a Gaussian probability, with an r.m.s. value given by [10]. The fluid turbulence intensity $\sqrt{\overline{v_f'^2}}$ is taken from Laufer's measurements in turbulent pipe flows (Hinze 1975). In this method, it is implicitly assumed that particle fluctuations are small: satisfactory results were obtained in dilute gas–solid flows when particles had a sufficiently high inertia for the ratio $V_f t^*/R$ to be much higher than unity, R being the pipe radius. This condition is generally satisfied in gas–solid flows with particles larger than 0.1 mm: let us consider, for instance, the case of 0.2 mm particles, of density 2600 kg/m^3, conveyed by air at 20 m/s in a 100 mm i.d. pipe. The particle relaxation time will be about 0.15 s, leading to a ratio $V_f t^*/R$ of the order of 60. The corresponding ratio of particle to fluid r.m.s. velocity, according to [10], will be about 4%. Such weak fluctuations of the particle velocity can be simulated satisfactorily by our approximate method. It must be emphasized that [10] gives only

the fluctuation due to fluid turbulence, bearing in mind that the fluctuation of the average solid phase velocity may be higher due to particle-to-wall and particle-to-particle collisions.

In the examples presented here, the influence of the particles on the fluid velocity profile was neglected (one-way simulation), and the gas velocity was assumed to obey the universal logarithmic law. This assumption, which could seem somewhat unrealistic in the case of non-dilute flows, was made for simplicity—since the main purpose of this work is the description of an original simulation method for particle-to-particle interactions. This method is suitable for all kinds of Lagrangian simulations, and can be described through a one-way simulation without loss of generality.

3. SIMULATION OF PARTICLE-TO-PARTICLE INTERACTIONS

3.1. Simulation principle

During the calculation of an individual trajectory, the occurrence of possible collisions with other particles is simulated by introducing some occasional changes in the velocity components, according to some well-defined rules which are described below. The number of collisions undergone by a particle is proportional to the number concentration of the surrounding particles. Therefore, the total number of collisions in the dispersed phase is proportional to the square of the concentration, and this is the reason why the influence of the corresponding discontinuities in the particle trajectories should not be underestimated when the concentration increases. Marble (1964) and Crowe (1981) discussed the importance and the mechanism of particle-to-particle collisions in a one-dimensional configuration. In the present work, the classical expression of the collision frequency was modified in order to account for the three-dimensional aspect of gas–solid flow. From analogy with the kinetic theory of gases, the frequency of collisions undergone by a particle of radius a_1 and velocity $\mathbf{V}_1$ is written as

$$\nu = \sqrt{2}\pi(a_1 + a_0)^2 \| \mathbf{V}_1 - \mathbf{V}_0 \| N, \qquad [11]$$

where the subscript 0 refers to the surrounding particles and N is the concentration (number/m^3). The factor $\sqrt{2}$ accounts for the difference between the one-dimensional situation and the three-dimensional one, with the assumption of a Maxwellian distribution of the particle relative velocity with respect to the local average solid phase velocity [see, for instance, Jeans (1954)].

The fraction of particles which will suffer a collision in the very small time interval $\mathrm{d}t$ is obviously $\nu\,\mathrm{d}t$. That is to say, the probability for a collision to occur in the time interval $\mathrm{d}t$ is $\nu\,\mathrm{d}t$. Now, let $\psi(t_0, t)$ be the probability for a particle to undergo a collision between instants t_0 and t. Hence, the probability for the same particle to travel during the time interval $t - t_0$ without colliding with another particle is

$$1 - \psi(t_0, t), \qquad [12]$$

therefore the probability for such a particle to experience a collision between instants t and $t + \mathrm{d}t$ is

$$\mathrm{d}\psi = \psi(t_0, t + \mathrm{d}t) - \psi(t_0, t) = [1 - \psi(t_0, t)]\nu\,\mathrm{d}t. \qquad [13]$$

Since $\psi(t_0, t_0) = 0$, integration of [13] yields:

$$\psi(t_0, t) = 1 - \exp[-\nu(t - t_0)]. \qquad [14]$$

We can conclude that $\psi(t_0, t)$ depends only on the difference $t_0 - t = \Delta t$, thus the probability $\Psi(\Delta t)$ for a collision to take place between instants t and $t + \Delta t$ is given by

$$\Psi(\Delta t) = \psi(t_0, t_0 + \Delta t) = 1 - \exp(-\nu\,\Delta t). \qquad [15]$$

This probability depends on both the local concentration N and the local average velocity $\mathbf{V}_0$ of the solid phase, via the collision frequency given by [11]. In order to be able to compute this probability at each point of a particle trajectory, it is necessary to store the values of N and $\mathbf{V}_0$ obtained from previous trajectory calculations. The flow domain is divided into a large number of cells, and the values of N and $\mathbf{V}_0$ in each cell are regularly updated in the corresponding arrays

by an iterative process (which is repeated until convergence is achieved). These arrays are initialized by means of a first simulation, which is performed without particle-to-particle collisions. Note that the simulated concentration, given by the number of particles obtained by the simulation in each cell, and thus dependent on the number of computed trajectories, must be corrected in order to obtain the corresponding actual concentration N, which is used for the prediction of the collision probability: the correction factor depends on the loading ratio and on the total number of simulated trajectories.

At each time step during a trajectory computation, the occurrence of a collision is decided by generating a random number with a uniform distribution between 0 and 1, and comparing it with the probability Ψ. In the case of collision, it is considered that the travelling particle (particle P_1) hits (or is hit by) a "standard" particle (particle P_0) which moves with the local average solid phase velocity $\mathbf{V}_0$. Furthermore, the particle P_0 is assumed to rotate with the local average angular velocity $\mathbf{\Omega}_0$, which must therefore be stored as well as N and $\mathbf{V}_0$. The position of the centre of the impacting particle P_1 is chosen randomly, with a uniform probability on the disk area of radius $a_0 + a_1$, orthogonal to the relative velocity (see figure 1).

3.2. Analysis of the collision dynamics

A detailed three-dimensional analysis of the collision between two translating and rotating spherical particles was presented by Lun & Savage (1987): both particles were assumed to be of the same diameter and mass, and the rotational aspect of the collision was modelled in an approximate way by means of a "roughness" coefficient. In another paper, Lun & Savage (1986) introduced an impact velocity dependent coefficient of restitution in their granular flow kinetic theory. In the present analysis, we describe the collision dynamics of two different spheres, of masses m_0 and m_1, and radii a_0 and a_1, moving at velocities $\mathbf{V}_0$ and $\mathbf{V}_1$ and rotating at angular velocities $\mathbf{\Omega}_0$ and $\mathbf{\Omega}_1$, by solving the complete impact equations under the sole assumption of a very small contact surface. We are mainly interested by the results concerning the unknown linear and angular velocity components after impact. It is preferable to study the collision dynamics in a reference frame moving with velocity $\mathbf{V}_0$, with the x-axis parallel to the centerline, and the y-axis in the plane defined by $\mathbf{V}_1$–$\mathbf{V}_0$ and x (as shown by figure 1), so that $V_{1z} - V_{0z} = 0$. Let a prime denote the linear and angular velocities just after impact. By applying the linear and angular momentum conservation laws to particle P_1, one obtains:

$$\left.\begin{aligned} V'_{1x} &= V_{1x} + \frac{J_x}{m_1} \\ V'_{1y} &= V_{1y} + \frac{J_y}{m_1} \\ V'_{1z} &= V_{1z} + \frac{J_z}{m_1} \\ \Omega'_{1x} &= \Omega_{1x} \\ \Omega'_{1y} &= \Omega_{1y} - \frac{5J_z}{2m_1 a_1} \\ \Omega'_{1z} &= \Omega_{1z} + \frac{5J_y}{2m_1 a_1} \end{aligned}\right\}, \qquad [16]$$

where m_1 is the mass of the particle P_1, a_1 is its radius and J_x, J_y, J_z are the components of the impulse $\mathbf{J} = m_1(\mathbf{V}'_1 - \mathbf{V}_1)$.

The definition of the coefficient of restitution e, namely

$$V'_{1x} - V'_{0x} = -e(V_{1x} - V_{0x}), \qquad [17]$$

and the x-momentum conservation of particle P_0 (mass m_0),

$$J_x = -m_0(V'_{0x} - V_{0x}), \qquad [18]$$

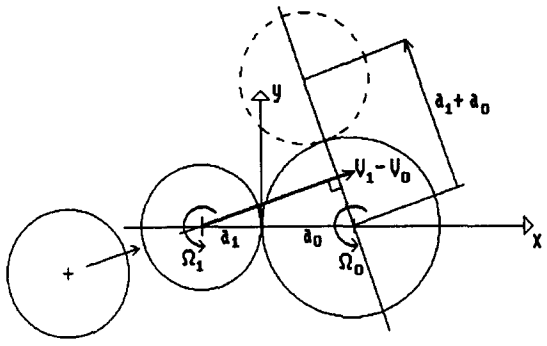

Figure 1. Particle-to-particle collision configuration.

lead to the expression of J_x in terms of $V_{1x} - V_{0x}$:

$$J_x = -(1+e)(V_{1x} - V_{0x})\frac{m_0 m_1}{m_0 + m_1}, \qquad [19]$$

whereas expressions for J_y and J_z depend on the nature of the impaction: as in the case of particle-to-wall collision, application of Coulomb's law yields the non-sliding condition in terms of the coefficient of static friction f_s:

$$\sqrt{J_y^2 + J_z^2} < f_s |J_x|. \qquad [20]$$

In order to write this condition in terms of the velocity components before impact, it is first assumed that the particle does not slide on the wall at the end of the collision, thus yielding expressions of J_y and J_z in terms of the known velocity components. After replacing in [20], one obtains the following non-sliding condition:

$$2W < 7f_s(1+e)|V_{1x} - V_{0x}|, \qquad [21]$$

where

$$W = \sqrt{W_y^2 + W_z^2}, \qquad [22]$$

$$W_y = V_{1y} - V_{0y} + a_1\Omega_{1z} + a_0\Omega_{0z} \qquad [23]$$

and

$$W_z = -a_1\Omega_{1y} - a_0\Omega_{0y}. \qquad [24]$$

If condition [21] is fulfilled, sliding ceases before the end of the collision time, and J_y and J_z are found to be

$$J_y = -\left(\frac{2}{7}\right)W_y\frac{m_0 m_1}{m_0 + m_1} \qquad [25]$$

and

$$J_z = -\left(\frac{2}{7}\right)W_z\frac{m_0 m_1}{m_0 + m_1}. \qquad [26]$$

In the opposite case, sliding remains during the whole collision time, and the tangential impulse components are proportional to J_x:

$$J_y = -f_k|J_x|\frac{W_y}{W} \qquad [27]$$

and

$$J_z = -f_k|J_x|\frac{W_z}{W}; \qquad [28]$$

f_k being the coefficient of kinetic friction.

These results make it possible to compute the new velocity components by use of [16], and thus to reinitialize the trajectory calculation after a particle-to-particle collision.

4. PRESSURE DROP PREDICTION

The pressure drop of the gas–solid flow between two given sections of the pipe can be obtained from the results of the simulation, by means of an axial momentum balance performed over the fluid–solid system. In accordance with the one-way coupling hypothesis, it is assumed that the fluid wall shear stress is not altered by the particulate phase. Thus, the total pressure drop can be considered as the sum of the pure gas flow pressure drop and of an additional pressure drop due to the solid particles. If the flow is fully developed, this additional pressure drop is balanced by the axial momentum losses during particle-to-wall collisions, which are the only exchanges which have to be taken into account, since particle-to-fluid and particle-to-particle interactions correspond to internal momentum exchanges for the whole fluid–solid system. Note that a solid phase momentum balance would show that these particle-to-wall momentum losses are balanced by the sum of the external forces acting on the particulate phase, which is a well-known result. That is to say, particle-to-particle interactions play an indirect role in the additional pressure drop, in modifying the concentration and velocity profile, and thus changing the wall collision number and conditions. If the flow is not fully developed, the calculation would involve, in addition, an integration of momentum fluxes in the entrance and exit sections.

Such a calculation, which requires the summation of wall collision axial momentum losses, was performed in the case of our horizontal pipe flow example, yielding the predicted pressure drop examined in the following section.

5. RESULTS

This new Lagrangian simulation technique was used for the prediction of a horizontal gas–solid pipe flow, with a loading ratio upto 20. Although such high loading ratios probably go beyond the validity of the one-way coupling hypothesis, it may be interesting to test the ability of the particle interaction model with increasing solid phase concentration, provided that particles do not deposit at the bottom of the pipe (since the applied particle-to-wall collision model does not hold if deposition occurs).

The length of the pipe was 10 m. Particles were assumed to enter the pipe with zero axial and rotational velocities. The initial transverse velocity of the particles, and their location in the inlet section, were chosen randomly in order to reproduce a Gaussian transverse velocity distribution and a uniform concentration profile. The other conditions were:

Pipe dia: 30 mm
Particle granulometry (Gaussian distribution): mean dia = 0.10 mm
standard deviation = 0.02 mm
Particle density: 2620 kg/m^3
Fluid (air): density = 1.17 kg/m^3
viscosity = $1.8 \cdot 10^{-5}$ Pa · s
mean velocity = $V_f = 25.5$ m/s
Particle-to-wall collisions: $f_k = 0.53$, $f_s = 0.68$, $e = 0.90$
Particle-to-particle collisions: $f_k = 0.40$, $f_s = 0.94$, $e = 0.95$
[Coefficients of friction are taken from the data of Fuller (1972), assuming glass particles and a copper pipe.]

Figure 2 shows the development of the vertical concentration profiles (normalized by the mean concentration) over the length of the pipe. The axial location is indicated by the ratio L/D (length over diameter of the pipe), recalling that the concentration was uniform in the inlet section ($L = 0$). The concentration profiles at loading ratios of 2, 5, 10 and 20 are compared with the results of the dilute suspension simulation, performed without interparticle collisions. An important influence of the particle-to-particle interactions is predicted by these results, which show a sensible alteration

of the concentration distributions even at moderate loading ratios. In spite of a general increase in concentration in the lower half of the duct, one can observe that the maximum concentration does not always occur at the lower wall. At a loading ratio of about 10, the concentration decreases in the neighbourhood of the lower wall. This phenomenon is possibly due to the existence of an equilibrium, at a distance from the wall, between the gravity effects and the rotation-induced lift effects, the latter being amplified by the collisions between particles. However, this effect was not observed at a loading ratio of 20, where the concentration was always a maximum at the bottom wall. No satisfactory explanation can be proposed for the moment. Furthermore, these results need experimental confirmation, which is very difficult to obtain because of the high concentration. Nevertheless, some concentration profile measurements in horizontal gas–solid pipe flow have been achieved at moderately high loadings; for example, by Morikawa *et al.* (1986), who investigated the flow of 0.4 mm polystyrene pellets suspended in an air stream at loading ratios upto about 10.

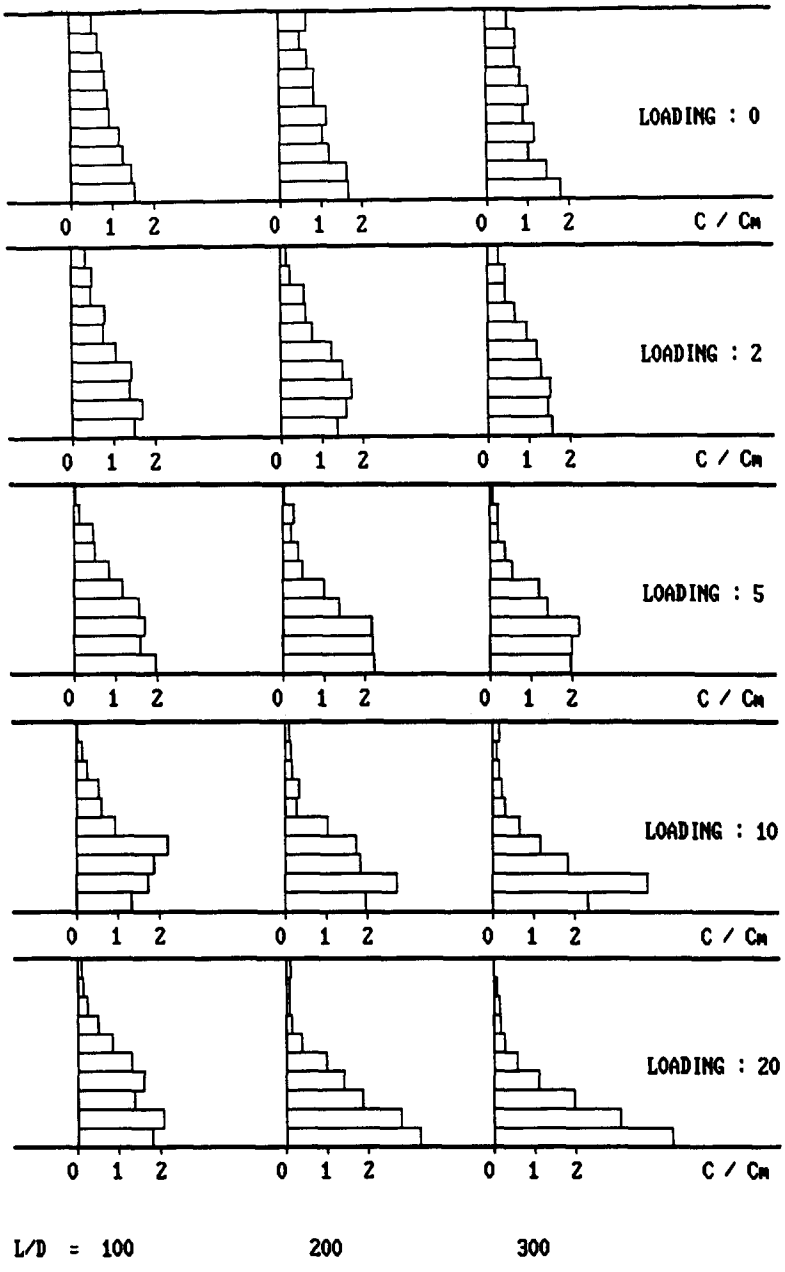

Figure 2. Development of vertical concentration profiles over the length of the pipe.

For velocities of 8–12 m/s and a pipe diameter of 52 mm, these authors observed the presence of deposition at loading ratios exceeding about 8. One may expect that this critical loading would increase with increasing air velocity. It would have been interesting to compare our simulation with these experimental results. Unfortunately, the values of the coefficients of friction and of restitution are not given in that paper, while our simulation results show an important influence of these coefficients, particularly for particle-to-wall friction (Oesterlé & Petitjean 1991). Furthermore, it seems that the particles used by Morikawa *et al.* (1986) were not spherical. Thus, the numerical simulation would only have been possible by using hypothetical values of the unknown coefficients, but the comparison would not have been significant.

The increase in the inlet length with increasing loading ratio is also apparent in figure 2, in considering the evolution of the concentration profiles. At small values of the loading ratio (0 or 2), no significant difference in the concentration profiles can be observed between $L/D = 100$, 200 or 300. At higher loading ratios, the stabilization of the concentration profile takes place at higher values of L/D: the dimensionless inlet length can be estimated at ~ 300 at the highest loading investigated here.

Unlike the concentration profiles, the solid phase velocity profiles seem to be only slightly influenced by increased loading, as can be seen in figure 3, which displays the vertical velocity distributions obtained in the fully developed region of the pipe at different loading ratios. The comparison with the dilute simulation result ("loading = 0") shows that these profiles are mainly altered in the bottom of the pipe, where the velocity diminishes as the concentration inceases. Such behaviour was also observed by Morikawa *et al.* (1986).

Pressure drops were computed in the fully developed region of the pipe (or quasi-developed, i.e. the last 2 m. Figure 4 is a plot of the pressure drop ratio $\Delta p/\Delta p_0$ vs the loading ratio (Δp_0 being the pure gas flow pressure drop). The results of the simulation are compared with previous measurements (Oesterlé 1978) corresponding to the above data, and with the expression derived by Michaelides (1987), based on a statistical analysis of a large number of experimental data obtained at moderate loadings. This expression reads

$$\frac{\Delta p}{\Delta p_0} = 1 + \frac{K}{\lambda}\frac{\dot{m}_p}{\dot{m}_f}\frac{\sqrt{gD}}{V_f}, \qquad [29]$$

where λ is the friction factor for the gas alone, defined as

$$\Delta p_0 = \frac{1}{2}\rho_f V_f^2 \lambda \frac{L}{D}, \qquad [30]$$

and the value of 0.041 was proposed by Michaelides (1987) for K in the case of glass particles.

Like many other correlations, this relationship indicates proportionality between the additional pressure drop and the loading ratio, because of neglecting interparticle effects in the model. It can be seen that our simulation, which is in good agreement with experimental data in the case of a dilute suspension, predicts a reduction in the influence of loading with increasing concentration. This fact is in accordance with other experimental results, like those of Farbar (1949) or Hitchcock & Jones (1958) for instance, and can be explained by the decreasing velocity in the neighbourhood

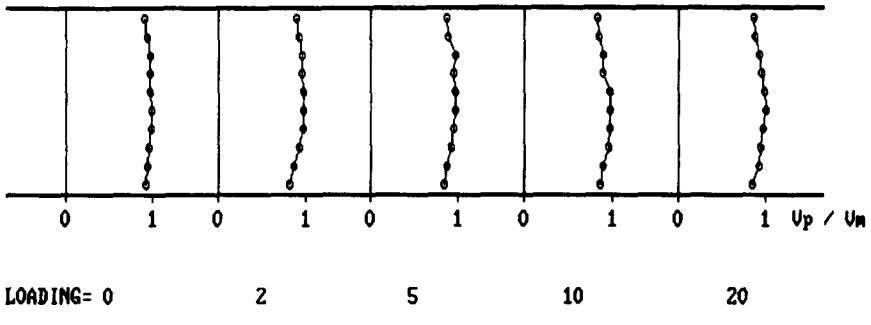

Figure 3. Vertical solid phase velocity profiles at $L/D = 300$.

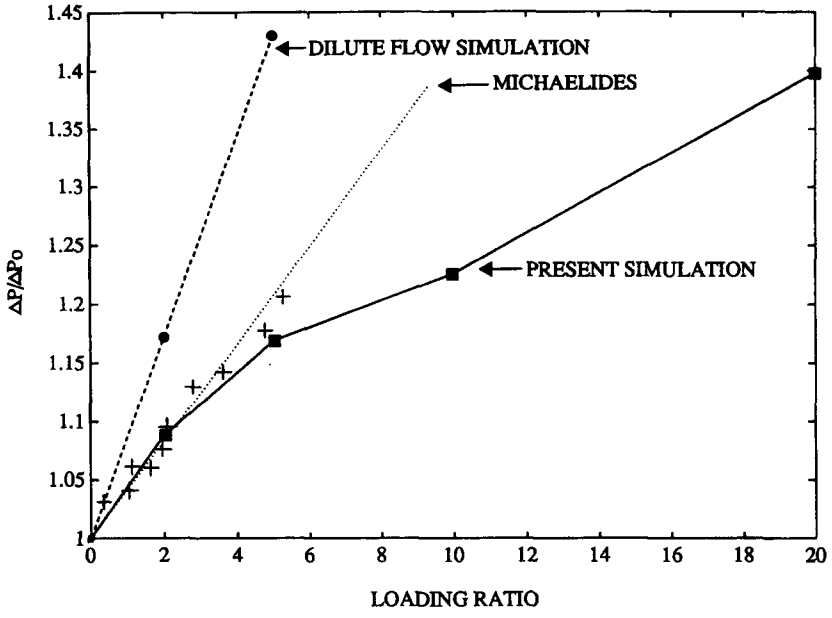

Figure 4. Pressure loss prediction in horizontal gas–solid pipe flow; +, experiments (Oesterlé 1978).

of the lower wall, where the number of particle-to-wall collisions is the largest, implying that the axial momentum losses are lower than would be predicted by a dilute flow simulation.

6. CONCLUSION

A new Lagrangian simulation technique, which enables the particle-to-particle collisions in gas–solid flows to be taken into account, was presented.

The main advantage of the method lies in the fact that dense or moderately dense flows can be simulated by computing a reasonable number of successive particle trajectories, contrary to previous simulations which required the computation of simultaneous trajectories and were limited by memory size.

In order to simulate particle-to-particle interactions, the detailed dynamics of the collision between two spherical particles was studied.

In spite of the lack of available exploitable experimental results, especially concerning concentration and velocity profiles at high loading ratios, pressure drop predictions obtained in horizontal pipe flow confirm that our simulation technique can yield useful information about industrial pneumatic transport. It must be pointed out that the results presented are only preliminary ones, and further tests have to be performed using a two-way coupling technique. The corresponding numerical code is suited to all kinds of pneumatic transport geometries, including the prediction of pressure drops due to acceleration and singularities.

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